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A CRITICAL ANALYSIS OF THE

METRI TECHNIQUE

by

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ABSTRACT

The "Military Essentiality Through Readiness Indices (METRI)" technique as presented in the Interim Technical Report to BuSandA, February 1965, was intended to describe the basic METRI model, the functional relationship between transition factors and the ship Readiness Index, as well as develop a technique for allowance list determination. However, many of the technical aspects of the report lacked clarity, some of the definitions were vague and doubly defined, and a sound theoretical justification was not adequately provided. The purpose of this thesis is to clarify and simplify certain technical aspects of METRI, provide theoretically sound models utilizing principles of reliability, and to derive the transition factors in a consistent and mathematically sound manner.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

1. Introduction.

Due to ever changing technology and the increasing complexity of weapons systems of our military services, methods of improved stocking and inventory policies for spare parts for these systems and their supporting systems have become mandatory. There have been a few scattered attempts in the literature to approach the inventory and allowance problem from a systems analysis point of view. The Bureau of Supplies and Accounts, in its continuing effort to provide the best and most reliable stocking and inventory policies possible, is always alert for new developments in this area. Research, analysis and implementation of procedures are continually being reviewed and revised to reflect the latest developments.

Quality control procedures were first devised to reflect the input of spare parts into the military supply systems. Sampling plans for both major equipment and repair parts were implemented. Once quality standards were established, some procedure had to be devised for determining which of these spares should become part of the on board allowance list for operating ships of the fleet, since it is not possible to stock a replacement for every item. Various techniques have been developed over the years and they are continually being replaced by new methods.

Recent developments indicate a tendency toward some classification of "essentiality" of a particular spare part as a criterion for establishing allowance lists. Some procedures require the review of each spare to establish its relative importance with respect to some

other particular part within the system. It has been pointed out [6] that one of the very important factors in the establishment of this essentiality or "Military Essentiality Code (MEC)" is the requirement of the "expert" or appropriate technical specialist to make both a judgement of the immediate application of the part as well as an estimate of its relative worth to the overall system. Clearly, this can be extremely difficult to accomplish. For example, although a fireman in an engine room may be quite familar with the operation, care, and replacement of burner nozzles for a burner for one of the boilers, it is unlikely that he would have a sufficient notion of the role played by that burner in the Anti-Submarine Warfare Mission of the ship to be able to provide the essentiality estimates. In particular such judgements would be very subjective in nature and not necessarily consistent.

Indeed, few individuals are capable of comprehending the complex interactions of the many thousands of parts and equipments necessary to operate a naval vessel. As indicated in [4], if a method can be devised which meaningfully organizes the existing areas of information and judgement, it may be possible to further devise measures and allowance lists that improve military capability over those that now exist. The basic goal in devising such measures is to improve resource allocation or the probability that the appropriate part will be available when required.

In the spring of 1963, a project was initiated by the Navy to devise an acceptable allowance list technique which would provide

either the desired level of "readiness" for any given budgetary constraint or the constraint to provide a particular level of "readiness". The resulting technique has come to be known as "Military Essentiality Through Readiness Indices (METRI)".

2. Definitions.

The following terms appear repeatedly throughout this paper and require a definition for consistent interpretation:

a) Structure- a line network model or flow chart which represents functional relationships of a system and its units in terms of Readiness Indices, (a METRI Chart).

b) Level- within the METRI hierarchy (the line network) a structure can be broken down into successive sub-units until it reaches its component parts. Each successive breakdown is a level.

c) Structure Model- a description of a structure which is not capable of further subdivision at the particular level of the line network. There are four types:

1) Series

2) Supplemental

3) Alternative

4) Collateral

d) Readiness Index, denoted R_{ij}, where i refers to a level, j refers to the j^{th} unit of the i^{th} level - a numerical value in the range $0 \leq R_{ij} \leq 1$ which represents the degree of readiness (a measure of utility) of the unit to perform its assigned function. In the models utilized in this paper R_{ij} is the reliability of the unit.

e) Unit- an element which can be distinguished as a distinct entity, e.g., component, part, system, subassembly.

f) Transition Factor (T_u)- the value of a transformation or mapping which relates the change in readiness index of a given

structure to a change in readiness index of a sparable part, or any lower level unit.

g) Sparable part— a part which has a spare or replacement item that is capable or being carried on board the ship.

3. Assumptions.

The following assumptions are basic to the understanding of the derivations and concepts developed in the remaining sections of this paper:

a) A ship is deployed for a cruise of length T units of time with specified installed and spared parts. No outside support is to be available during time T.

b) The Readiness Index of a component, R_{ij}, is the reliability (measure of the probability that an unspared failure will not occur during the duration of the cruise) of that component.

c) The underlying failure distribution of the repair parts is assumed to be Poisson for each individual part.

d) If a particular spare part has m multiple applications, the failure distributions of each of the m applications are independent.

e) The failure rate for identical repair parts is considered to be a constant λ.

f) The Weibull distribution "burn-in", "burn-out" portions are considered to be averaged with all applications of the same spare part so that the overall effect is the average failure rate λ. Obviously, this is equivalent to assuming that failures are exponentially distributed with constant failure rate λ.

g) Secondary failure (induced failure) is excluded in the computations.

h) Repair or replacement time is negligible.

i) Cannibalization (the removal of functioning units from some inoperative equipment to replace a failed unit in an otherwise operable piece of equipment) does not take place.

4. The METRI Concept.

The METRI technique is designed to measure the "readiness" or the degree to which a complex system is capable of performing its assigned task. The variable which reflects the degree of readiness is the Readiness Index, a dimensionless quantity which varies from 0 to 1. The Readiness Index is applicable first as a measure of the over-all system effectiveness of the ship for use as a decision criterion for operational decisions, and secondly as a measure of the readiness of each unit of the total structure to perform its designed mission. This second application is the basis for the METRI structural models. The total system is partitioned into subsystems and the Readiness Index is expressed as a function of the degree of readiness of each of the subsystems. The method of partitioning proceeds through the total system in a type of hierarchal structure, a composition beginning with the ship, its missions, the equipment necessary to accomplish each mission, the component subsystems comprising each piece of equipment necessary to a mission, and so on down to the individual part level. For the symbolic representation of this concept see Section 6, page 28.

Reference [2] states:

> The index has been developed to treat the problem of evaluating the degree to which a job may be performed. It is not concerned with how this degradation (sic) occurs. It may result from failure and poor maintenance, stock-out of repair parts, poor operation, etc. The question of how becomes important only in taking corrective action. The readiness meter must respond equally regardless of cause.
>
> Once the model is developed and the necessary data become available, the question of how this measuring system

is used arises. Briefly, a readiness calculation is made for certain stock conditions and then varied to determine whether increases or decreases in stock give reasonable changes in readiness as compared to the change in cost of the associated repair parts. New allowance lists may be calculated resulting from changes in management policy and these evaluated by computing the resulting ship readiness and inventory costs.

In order to reflect the change induced by increasing or decreasing stock level, a method of expressing the change in the readiness index of the ship due to a change in the number of parts on board has been derived. The incremental change in ship readiness is reflected by means of the Transition Factor (T_u) applied to readiness change at other levels. The equation expressing the change in readiness of a part due to addition of a spare has also been developed.

The basic concept of the allowance list determination is to calculate the change in the Readiness Index of the ship due to the addition of the k^{th} spare for part i, $\Delta R_s(k_i)$. The essentiality of this k^{th} spare is then defined to be this change in Readiness Index divided by the price of the spare (p_i). Parts will then be selected on the basis of essentiality until some overall readiness is attained, or else some budgetary constraint is satisfied. Other constraints are possible, but will not be considered, e.g., cube, weight, etc. Thus the objective is to determine the number of parts (n_i) which will maximize the Ship Readiness Index (R_s) subject to a budget constraint (C), expressed as

$$\sum_i n_i p_i \leq C$$ where each index i refers to a sparable item

Having defined the METRI concept, the basic purpose of the Interim Technical Report [2] was to amplify on the basic elements of the model and describe the functional relationship between Transition Factors and the Ship Readiness Index. Moreover, from such an analysis a technique for establishing allowance lists was to be established. However, many of the technical aspects of the above Interim Report lacked clarity, definitions tended to be vague and circular, and in some cases, lacked sound theoretical justification. These and other critical matters became the main concern of several document reviews ([11] through [16], [20] through [22]).

The purpose of this thesis is to clarify and simplify certain technical aspects of METRI by carefully structuring reliability models of some of the basic concepts documented in [2]. Once appropriate models are defined, it is then possible to derive the Transition Factors and their role in ship readiness in a consistent and mathematically sound manner. Such an analysis is presented in the ensuing sections.

5. Models.

a. Series model. The units of a series model are linked together in a linear sequence as depicted in Figure 1. The effects of the individual unit readiness indicies are multiplicative, the heuristic argument being that they function as a whole, and if one unit fails the entire sequence fails. The mathematical model of such circumstances leads to the theory of Series Reliability models.

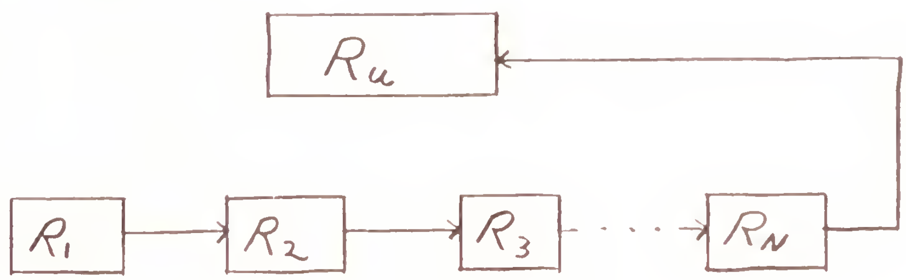


Figure 1. Series Model

Equation 1. $R_u = R_1 R_2 \cdots R_N$

where R_u is the Series Model Readiness Index

R_i is the individual component Readiness Index, $i = 1, 2, \cdots, N$

Example: A series circuit in a radio is composed of five tubes in series. If one of the tubes burns out with no spare available the series circuit fails.

b. Supplement model. The units of a supplement model affect the model in an additive manner and failure of any particular one does not necessarily cause a failure of the entire model as depicted in Figure 2. Each unit contributes a percentage of the total readiness index. Heuristically, the failure of a unit does not cause the model to

fail but can degrade the overall readiness index. The determination of the degradation factor (K factor) will be mentioned later in the paper.

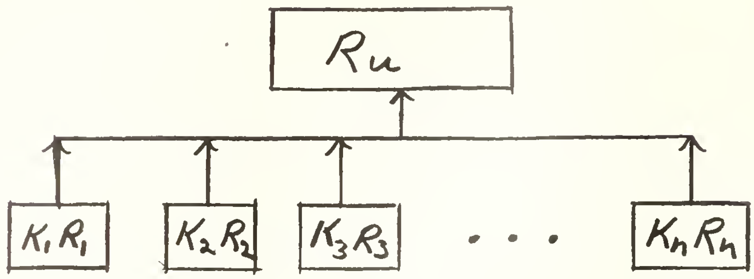


Figure 2. Supplemental Model

Equation 2. $R_u = K_1R_1 + K_2R_2 + \cdots + K_nR_n = \sum_{i=1}^{n} K_iR_i$

where R_u is the supplement model Readiness Index

R_i is the individual unit Readiness Index $i = 1, 2, \cdots, n$

K_1 is the individual degradation factor $i = 1, 2, \cdots, n$

and we suppose that $\sum_{i=1}^{n} K_i = 1$.

Example: An internal combustion engine with spark plugs serves to exemplify the Supplemental Model. Each plug provides a percentage of the total output. The failure of any one plug does not cause the engine to fail. However the rated output is degraded and if all of the spark plugs were to fail without replacement the engine would fail.

c. Alternate model. The alternate model reflects the readiness index of a system composed of two units, where the alternate unit can perform the functions of the primary unit, but with some possible degradation of the readiness index of the alternate unit.

Suppose that

K_2 represents the degradation factor for the alternate unit,

S_u is the event that Unit u is successful (does not fail),

A_1 is the event that the primary unit works (does not fail),

$\overline{A}_1$ is the event that the primary unit fails (does not work),

A_2 is the event that the alternate unit works.

Since $S_u = A_1 \cup A_2 = A_1 \cup (A_2 - A_1) = A_1 \cup (A_2 \cap \overline{A}_1)$,

$P\left[S_u\right] = P\left[A_1\right] + P\left[A_2 | \overline{A}_1\right] P\left[\overline{A}_1\right]$

where $P\left[S_u\right]$ is the probability of event S_u,

$P\left[A_1\right]$ is the probability of event A_1 and it is assumed that

$P\left[A_2 | \overline{A}_1\right] = K_2 R_2$ with $0 \leq K_2 \leq 1$.

Thus $R_u = R_1 + K_2 R_2 \left[1 - R_1\right]$ in terms of reliabilities or readiness indices.

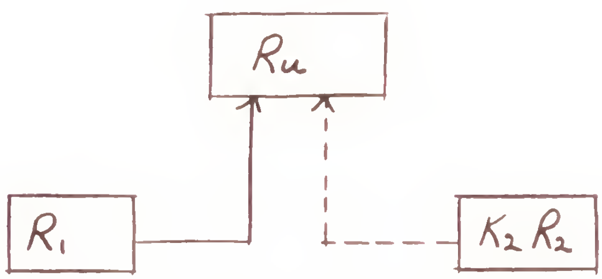


Figure 3. Alternate Model

Equation 3. $R_u = R_1 + K_2 R_2 \left[1 - R_1\right]$

Example: The ships main generator for electric power is the primary unit. A diesel powered generator provides auxiliary power. In case of emergency or failure of the primary generator the diesel generator can provide main power but in a degraded (less power)

rating.

d. Collateral model. Occassionally, at a given level, a nonessential unit may enter into the model in the sense of contributing a "nice to have" factor to the readiness index (degree of readiness) of the essential unit, say, by facilitating usage. When this is the case, it is necessary to account for this factor and the corresponding model is called the Collateral model. However, it is important to note that failure of the nonessential unit would <u>not</u> cause failure at that level. Consequently, when a change in the nonessential unit is not a consideration, the readiness index at this level is the readiness index of the essential unit and the collateral portion of the model plays no role.

Let S_u be the event that the unit works,

A_1 the event that the essential element works and

A_2 the event that the nonessential element works.

Then, $S_u = A_1 = (A_1 \cap A_2) \cup (A_1 \cap \bar{A}_2)$, so that

$$P[S_u] = P[A_1 \cap A_2] + P[A_1 \cap \bar{A}_2] \text{ or,}$$

$$P[S_u] = P[A_1] = P[A_1 | A_2]\, P[A_2] + P[A_1 | \bar{A}_2]\, P[\bar{A}_2].$$

The contribution of the nonessential unit to the readiness of the system is indicated by a "conditional reliability", namely, we suppose that

$$R_1^* = P[A_1 | A_2] \text{ and}$$

$K_2 R_1^* = P[A_1 | \bar{A}_2]$ where K_2 is a degradation factor, $0 \leq K_2 \leq 1$.

Let $R_2 = P[A_2]$ be the reliability of the nonessential unit and

$R_1 = P[A_1]$ be the reliability of the essential unit (which is the same as unit reliability, R_u, in this case).

Then, the above probability statement can be expressed in terms of reliabilities as follows,

$$R_u = R_1^* R_2 + K_2 R_1^* (1-R_2)$$
$$= R_1^* [K_2 + (1-K_2) R_2].$$

Figure 4. Collateral Model

Equation 4. $R_u = R_1^* [K_2 + (1-K_2) R_2]$

Example. The air filter system to a diesel provides clean air for combustion and serves to exemplify the collateral model. The engine will run without the air filter but it is receiving impurity particles which degrade performance.

e. Availability model. The Interim Technical Report [2] specifies the need for an availability model and further indicates three ways in which availability is to be defined:

> (1) Instantaneous Availability- The probability that an equipment will be available (i.e., in an operating state at any random time T).
> (2) Average Up-time: The proportion of time in a specified interval (0,T) that an equipment is available for use.

(3) Steady State Availability: The proportion of time that an equipment is available for use when the time interval considered is very large.

Availability is thus a function of how often an equipment fails and how long it takes to restore the equipment once it has failed.

It is contended that (1) above is merely a restatement of the probability that an unspared failure does not occur which is the reliability. In the case of (2) and (3), since the basic assumption was that of an unsupported ship, the actual time to repair a particular system is relatively minute with respect to the total deployed time, given that a spare part was available. Therefore, the system is in effect a sub-category of availability as defined in (1) which, as previously stated, is reliability. Otherwise the system is not reparable and would be lost during the remainder of the cruise. In effect, then, the Availability model is superfluous.

6. Transition Factors (T_u).

Transition factors are required in order to facilitate the computation of the incremental change in the highest unit Readiness Index. The METRI structuring technique exhibits the explicit relationships which exist between the highest unit and any particular unit at some lower level. This can be found in particular between the highest unit and a component part at the lowest possible individual repair part level since there always exists a path from the highest unit to some individual sparable part.

Once all paths have been identified and labeled, the effect of any given lower level component on the overall system can be determined.

The following paragraphs develop a general concept of the functional relationships necessary to derive the Transition Factor, which will be subsequently considered.

Consider the following functions:

1) $z= f(u,v)$

2) $u= g_1(w,r)$

3) $v= g_2(x,y)$

4) $w= h_1(s,t)$

5) $r= h_2(p,q)$

6) $x= h_3(m,n)$

7) $y= h_4(k,l)$

where f, g_i $i=1,2$; h_j, $j=1,\cdots,4$ are functions of the variables indicated. Then z may be expressed as follows:

8) $z = f_1(w,r,x,y)$ where f_1 is some function of the variables indicated.

9) $z = f_2(k,l,m,n,p,q,s,t)$ where f_2 is some function of the variables indicated.

Taking the total derivatives of equations 1 through 9:

10) $dz = \frac{\partial f}{\partial u} du + \frac{\partial f}{\partial v} dv$

11) $du = \frac{\partial g_1}{\partial w} dw + \frac{\partial g_1}{\partial r} dr$

12) $dv = \frac{\partial g_2}{\partial x} dx + \frac{\partial g_2}{\partial y} dy$

13) $dw = \frac{\partial h_1}{\partial s} ds + \frac{\partial h_1}{\partial t} dt$

14) $dr = \frac{\partial h_2}{\partial p} dp + \frac{\partial h_2}{\partial q} dq$

15) $dx = \frac{\partial h_3}{\partial m} dm + \frac{\partial h_3}{\partial n} dn$

16) $dy = \frac{\partial h_4}{\partial k} dk + \frac{\partial h_4}{\partial l} dl$

17) $dz = \frac{\partial f_1}{\partial w} dw + \frac{\partial f_1}{\partial r} dr + \frac{\partial f_1}{\partial x} dx + \frac{\partial f_1}{\partial y} dy$

18) $dz = \frac{\partial f_2}{\partial k} dk + \frac{\partial f_2}{\partial l} dl + \frac{\partial f_2}{\partial m} dm + \frac{\partial f_2}{\partial n} dn + \frac{\partial f_2}{\partial p} dp + \frac{\partial f_2}{\partial q} dq$
$+ \frac{\partial f_2}{\partial s} ds + \frac{\partial f_2}{\partial t} dt$

Now substituting 11 and 12 into 10)

19) $dz = \frac{\partial f}{\partial u}\left[\frac{\partial g_1}{\partial w} dw + \frac{\partial g_1}{\partial r} dr\right] + \frac{\partial f}{\partial v}\left[\frac{\partial g_2}{\partial x} dx + \frac{\partial g_2}{\partial y} dy\right]$

comparable substitutions can be made in equation 19) for dw, dr, dx and dy so that

$$20)\ dz = \frac{\partial f}{\partial u}\left[\frac{\partial g_1}{\partial w}\left(\frac{\partial h_1}{\partial s}ds + \frac{\partial h_1}{\partial t}dt\right) + \frac{\partial g_1}{\partial r}\left(\frac{\partial h_2}{\partial p}dp + \frac{\partial h_2}{\partial q}dq\right)\right] +$$

$$\frac{\partial f}{\partial v}\left[\frac{\partial g_2}{\partial x}\left(\frac{\partial h_3}{\partial m}dm + \frac{\partial h_3}{\partial n}dn\right) + \frac{\partial g_2}{\partial y}\left(\frac{\partial h_4}{\partial k}dk + \frac{\partial h_4}{\partial l}dl\right)\right]$$

However, given that all of the differentials of equation 18) except one, say dt, are zero, then, providing $dt > 0$;

$dy = 0$ since $dk = dl = 0$

$dx = 0$ since $dm = dn = 0$

$dr = 0$ since $dp = dq = 0$

$dw = \frac{\partial h_1}{\partial t}dt$ since $ds = 0$

$dv = 0$ since $dx = dy = 0$

$du = \frac{\partial g_1}{\partial w}dw$ since $dr = 0$

Making substitutions in equations 10 through 18)

$$10A)\ dz = \frac{\partial f}{\partial u}du$$

$$11A)\ du = \frac{\partial g_1}{\partial w}dw$$

$$13A)\ dw = \frac{\partial h_1}{\partial t}dt$$

$$17A)\ dz = \frac{\partial f_1}{\partial w}dw$$

$$18A)\ dz = \frac{\partial f_2}{\partial t}dt$$

$$19A)\ dz = \frac{\partial f}{\partial u}\left[\frac{\partial g_1}{\partial w}dw\right]$$

$$20A)\ dz = \frac{\partial f}{\partial u}\left[\frac{\partial g_1}{\partial w}\left(\frac{\partial h_1}{\partial t}dt\right)\right] = \frac{\partial f}{\partial u}\frac{\partial g_1}{\partial w}\frac{\partial h_1}{\partial t}dt$$

Let the symbol T_α represent the functional relationship between dz and the variable α, given that all differentials except $d\alpha$ are zero.

Thus, $10A^1$) $dz = \frac{\partial f}{\partial u} du = T_u du$ where $\frac{\partial f}{\partial u} = T_u$

$17A^1$) $dz = \frac{\partial f_1}{\partial w} dw = T_w dw$ where $\frac{\partial f_1}{\partial w} = T_w$

$18A^1$) $dz = \frac{\partial f_2}{\partial t} dt = T_t dt$ where $\frac{\partial f_2}{\partial t} = T_t$

but also $19A^1$) $dz = \frac{\partial f}{\partial u}\frac{\partial g_1}{\partial w} dw$ which implies $T_w = \frac{\partial f}{\partial u}\frac{\partial g_1}{\partial w}$ since

$dz = T_w dw$

and $20A^1$) $dz = \frac{\partial f}{\partial u}\frac{\partial g_1}{\partial w}\frac{\partial h_1}{\partial t} dt$ which implies $T_t = \frac{\partial f}{\partial u}\frac{\partial g_1}{\partial w}\frac{\partial h_1}{\partial t} dt$

since $dz = T_t dt$

This further implies, under the assumption $di=0$ $i=k,l,m,n,p,q,s$, that (by equating the partial derivative terms of $17A^1$ and $19A^1$)

$$\frac{\partial f_1}{\partial w} = \frac{\partial f}{\partial u}\frac{\partial g_1}{\partial w}$$

which says that the partial of f_1 with respect to w(given dr=dx=dy=0) is equal to the partial derivative of f(u,v) with respect to u (given dv=0) multiplied by the partial derivative of g_1 with respect to w(given dr=0).

Also by equating the partial derivative terms of $18A^1$ and $20A^1$) it is likewise true that

$$\frac{\partial f_2}{\partial t} = \frac{\partial f}{\partial u}\frac{\partial g_1}{\partial w}\frac{\partial h_1}{\partial t}$$

which is nothing more than the additional product of the partial derivative of h_1 with respect to t(given that ds=0) for the total variable functional expression f_2.

That is to say, that by holding all variables constant except

for the one of interest, (dt in example), a functional relationship consisting of the product of partial derivatives of successive relationships ($\frac{\partial f}{\partial u} \cdot \frac{\partial g_1}{\partial w} \cdot \frac{\partial h_1}{\partial t}$ in this example) is equivalent to the partial derivative ($\frac{\partial f_2}{\partial t}$).

In terms of the T_α notation, note that by 10A[1], 17A[1] and 11)

$$dz = T_u du = T_w dw$$

But by 19A) $\quad dz = T_u \frac{\partial g_1}{\partial w} dw$,

so that $\quad T_w = T_u \frac{\partial g_1}{\partial w}$

That is to say that T_w (which is a relationship between dz and the partial derivative of the function f_1 (w,r,x,y) where dr=dx=dy=0) is equal to T_u (which is the partial derivative of h(u,v) with dv=0) multiplied by the partial derivative of g_1 (w,r) with dr=0 again a product of successive partial derivatives.

Likewise by 18A[1] , 17A[1] and 13)

$$dz = T_t dt = T_w dw$$

so that $\quad T_t = T_w \frac{\partial h_1}{\partial t}$

and substituting

$$T_w = T_u \frac{\partial g_1}{\partial w}$$

yields $\quad T_t = T_u \frac{\partial g_1}{\partial w} = \frac{\partial f}{\partial t} \frac{\partial g_1}{\partial w} \frac{\partial h_1}{\partial t}$,

which again is the successive product of partial differentials where all variables except one are held constant.

In terms of the METRI symbols let

R_{ij} be the readiness index of the ij^{th} unit, where

i represents the level,

j represents the j^{th} unit of the i^{th} level,

$i = 1,2,\cdots,N$ (N is the number of levels)

Note that $j=0$ when $i=1$; otherwise $1 \leq j \leq n_i$

We will let R_{10} represent the top structure so that

$$R_{10} = f_1(R_{2,1}, R_{2,2},\cdots,R_{2,n_2})$$

$$R_{2j} = f_{2j}(R_{3,1}, R_{3,2},\cdots,R_{3,n_3}) \quad j = 1,\cdots,n_2 \text{ and}$$

in general

$$R_{ij} = f_{ij}(R_{i+1,1}, R_{i+1,2},\cdots, R_{i+1,n_{i+1}}) \; j=1,2,\cdots,n_i$$

where the f_{ij} represents the METRI basic structure equation which expresses R_{ij} in terms of the $R_{i+1,k}$ $k=1,2,\cdots,n_{i+1}$.

Differentiating the R_{ij} expression, letting d represent the diffenential symbol,

$$dR_{ij} = \sum_{k=1}^{n_{i+1}} \frac{\partial f_i}{\partial R_{i+1,k}} dR_{i+1,k} \qquad k=1,2,\cdots,n_{i+1}$$

Letting all the $dR_{i+1}=0$, except the j^{th} unit $(dR_{i+1,j})$, then as previously shown

$$dR_{10} = T_{ij} dR_{ij} \quad \text{where } T_{ij} \text{ is the Transition Factor.}$$

Example. Reference Figure 5 and 6. The example structure is shown in Figure 5 and represents a dual mission ship. Figure 6 shows the

level breakdown and the applicable functional equations and total derivatives of each functional equation. See Appendix B for the derivation of the total derivatives of each of the METRI models.

1) $f_1 = K_{21}R_{21} + K_{22}R_{22}$

$$dR_{10} = K_{21}dR_{21} + K_{22}dR_{22}$$

and, letting $dR_{22} = 0$,

$$dR_{10} = K_{21}dR_{21}$$

$$T_{21} = K_{21} = \frac{\partial f_1}{\partial R_{21}}$$

2) $f_2 = R_{31} + K_{32}R_{32}(1-R_{31})$

$$dR_{21} = (1-K_{32}R_{32})\, dR_{31} + K_{32}\,(1-R_{31})\, dR_{32}$$

and, letting $dR_{32} = 0$,

$$dR_{21} = (1- K_{32}R_{32})dR_{31} \quad .$$

Substituting dR_{21} into dR_{10} ,

$$dR_{10} = K_{21}(\, 1-K_{32}R_{32})\, dR_{31} = T_{31}dR_{31}$$

where, $T_{31} = K_{21}(\, 1-K_{32}R_{32}) = \frac{\partial f_1}{\partial R_{21}} \cdot \frac{\partial f_2}{\partial R_{31}}$

3) $f_3 = f_4$

Notice that this is a Collateral model and since (in this example) we are not concerned with the nonessential unit, $f_3 = f_4$. Otherwise we would write

$$f_3 = R_{41}^{*}\left[K_{42} + (1 - K_{42})\, R_{42}\right] \quad ,$$

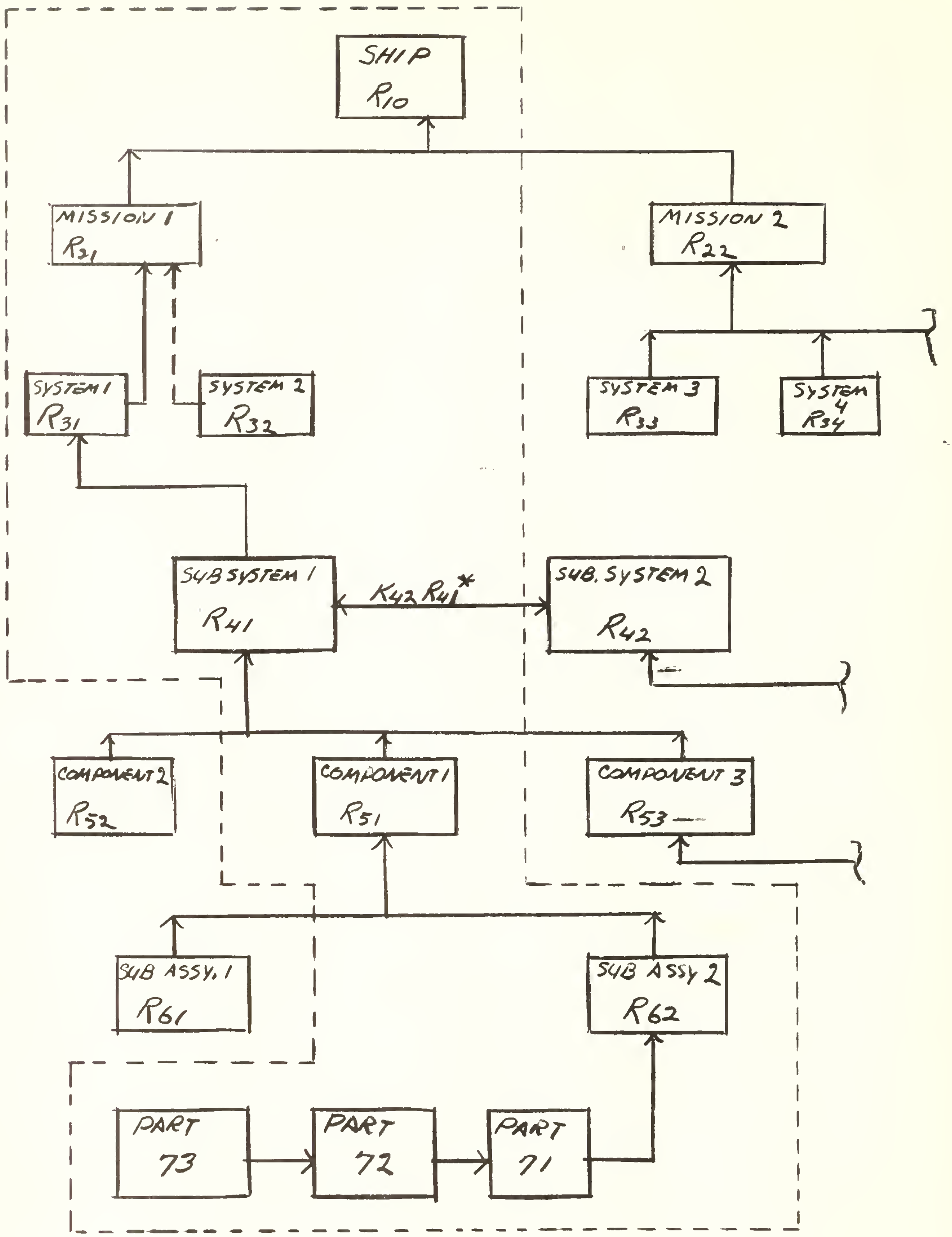
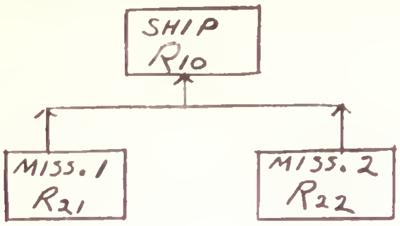


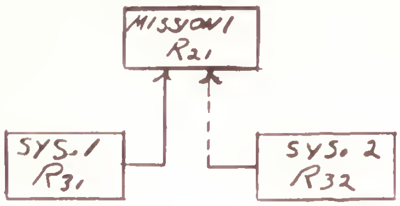
Figure 5.
A Dual mission ship with partial structure breakdown. See Fig.6. for area inclosed in dashed lines.



Supplement Model

$$R_{10} = K_{21}R_{21} + K_{22}R_{22}$$

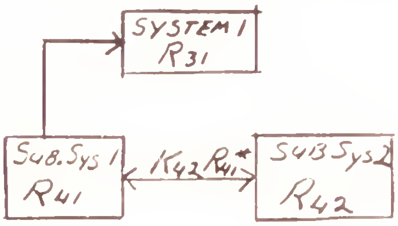
$$dR_{10} = K_{21}dR_{21} + K_{22}dR_{22}$$



Alternate Model

$$R_{21} = R_{31} + K_{32}R_{32}(1-R_{31})$$

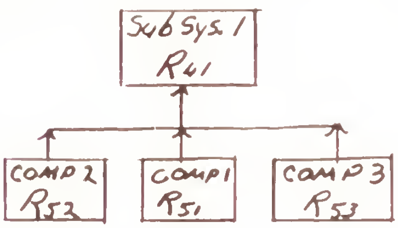
$$dR_{21} = (1-K_{32}R_{32})dR_{31} + K_{32}(1-R_{31})dR_{32}$$



Collateral Model

$$R_{31} = R_{41}^{*}\left[K_{42} + (1-K_{42})R_{42}\right]$$

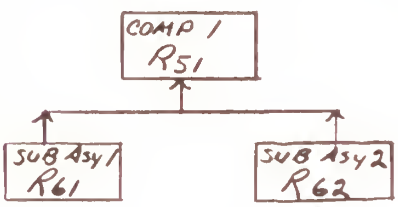
$$dR_{31} = \left[K_{42} + (1-K_{42})\right]dR_{41}^{*} + R_{41}^{*}(1-K_{42})dR_{42}$$



Supplement Model

$$R_{41} = K_{51}R_{51} + K_{52}R_{52} + K_{53}R_{53}$$

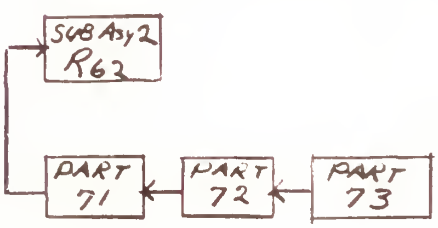
$$dR_{41} = K_{51}dR_{51} + K_{52}dR_{52} + K_{53}dR_{53}$$



Supplement Model

$$R_{51} = K_{61}R_{61} + K_{62}R_{62}$$

$$dR_{51} = K_{51}dR_{51} + K_{62}dR_{62}$$



Functional Relationship of R_{62} with its component parts (series in this case) $R_{62} = R_{72}R_{73}R_{71}$

$$dR_{62} = R_{72}R_{73}\ R_{71} + R_{71}R_{72}\ R_{73} + R_{71}R_{73}\ R_{72}$$

Figure 6.

in which case

$$dR_{31} = \left[K_{42} + (1-K_{42})R_{42}\right] dR_{41}^{*} + R_{41}^{*}(1-K_{42})dR_{42} \quad .$$

Also, from the transition factor derivation procedures, dR_1^* of the Collateral model will be set to zero when considering the effect of the nonessential unit. Moreover, when the effect of the nonessential unit is not being considered, R_1^*, and hence dR_1^*, never enter the formulas. Thus, in no case is it ever true that $dR_1^* > 0$. Continuing,

4) $f_4 = K_{51}R_{51} + K_{52}R_{52} + K_{53}R_{53}$

$$dR_{41} = K_{51}dR_{51} + K_{52}dR_{52} + K_{53}dR_{53}$$

and, letting $dR_{52} = dR_{53} = 0$,

$$dR_{41} = K_{51}dR_{51}.$$

Substituting,

$$dR_{10} = T_{41}dR_{41} = T_{41}K_{51}dR_{51}$$

$$T_{51} = T_{41}K_{51} = \frac{\partial f_1}{\partial R_{21}} \cdot \frac{\partial f_2}{\partial R_{31}} \cdot \frac{\partial f_4}{\partial R_{51}}$$

5) $f_5 = K_{61}R_{61} + K_{62}R_{62}$

$$dR_{51} = K_{61}dR_{61} + K_{62}dR_{62}$$

and, letting $dR_{61} = 0$,

$$dR_{51} = K_{62}dR_{62}.$$

Substituting,

$$dR_{10} = T_{51}dR_{51} = T_{51}K_{62}dR_{62}$$

$$T_{62} = T_{51}K_{62} = \frac{\partial f_1}{\partial R_{21}} \cdot \frac{\partial f_2}{\partial R_{31}} \cdot \frac{\partial f_4}{\partial R_{51}} \cdot \frac{\partial f_5}{\partial R_{61}}$$

6) dR_{62} is the change in readiness index of unit number 62 due to the change in readiness index of its component parts.

The change in readiness index of unit 62 due to the addition of a spare part is the functional relationship of that part to its components. The readiness index of the example part 72 is in sequence with parts 71 and 73. Therefore the change in readiness of part 72 due to the addition of the n+1st spare is given by $\Delta R_{72}(n+1)$, where in general,

(Equation 5)

$$\Delta R_j(n+1) = \frac{1}{m} \frac{e^{-m\lambda T} (m\lambda T)^{n+1}}{(n+1)!} + \frac{1}{m} \sum_{j=n+2}^{\infty} \left(\frac{m-1}{m}\right)^{j-n-1} \frac{(m\lambda T)^j e^{-m\lambda T}}{j!} .$$

Since

(Equation 6)

$$R_j(n) = \sum_{k=0}^{n} \frac{e^{-m\lambda T} (m\lambda T)^k}{k!} + \sum_{j=n+1}^{\infty} \left(\frac{m-1}{m}\right)^{j-n} \frac{e^{-m\lambda T} (m\lambda T)^j}{j!}$$

(See appendix A for derivation of the equation).

The change in the readiness index of the immediate parent assembly 62, dR_{62}, is computed by the use of equation 5 and the relationship of the parts to the component expression

$$dR_{62} = f_{62} \Delta R_{72}(n+1) = R_{71} R_{73} \Delta R_{72}(n+1)$$

where f is the functional relationship expressing a change in a parts readiness index to the effect on the parent component. (In the case of the example, $f_{62} = R_{71} R_{73}$)

Therefore the readiness of the ship can be expressed as a function of the change in readiness of the i^{th} part by the T-factor equations, for example

$$dR_{10} = T_u dR_u$$

$$dR_{10} = T_{62} dR_{62} = T_{62} f_{62} \Delta R_{72}(n+1)$$

The change in ships readiness index can then be expressed in terms of a change in the readiness index of a part. By dividing this change in readiness index by the cost of the part, the change in readiness index per dollar cost can be expressed. Then a maximumization problem can be formulated in terms of change in readiness index per dollar cost subject to a total cost constraint which maximizes a ships readiness index.

7. An Iterative Technique for Allowance List Determination.

Once the Transition factors, part readiness, and the incremental change in readiness index model have been calculated, the essentiality of a particular spare part can be determined. The change in readiness index of the top structure due to the addition of the $n+1^{st}$ spare part for a particular N^{th} level component is expressed, symbolically, as

$$dR_{10} = T_{ij} dR_{ij} \qquad i = 1, \cdots, N; \quad j = 1, 2, \cdots, n_i$$

$$= T_{Nj} dR_{Nj}$$

where N is the lowest level and j is the particular component under consideration. As shown in the last Section, dR_{Nj} can be represented as some functional relationship of all the individual sparable parts of the component multiplied by the change in readiness index for the $n+1^{st}$ spare added to support a particular sparable part of the component.

Symbolically,

$$dR_{Nj} = f_{Nj} \Delta R_{lm}(n+1),$$

where lm is the identification for the particular component.

Thus, $$dR_{10} = T_{Nj} f_{Nj} \Delta R_{lm}(n+1)$$

and, dividing dR_{10} by the cost of the $n+1^{st}$ spare, c_{lm}

$$E_{lm}(n+1) = \frac{dR_{10}}{c_{lm}}$$

where $E_{lm}(n+1)$ = essentiality of the $n+1^{st}$ spare for the lm^{th} part.

In order to initiate the technique, a beginning position must be realized. One possible position would be with all sparable parts installed and operating with no spare parts available. Making an assumption that, initially, the readiness index of each component with installed parts is required to be some predetermined amount, and also the readiness index of each sparable part is required to be some predetermined value, Equation 5 is used repeatedly until this part readiness is attained. For the METRI model in the example, Chapter 8 of [2],

$$R_{ij} = .9999999 \quad , \quad i=1,\cdots,N \; ; \quad j=1,\cdots,n_i$$

Since all R_{ij} are known for each level of the METRI structure, the Transition Factor, T_{ij}, for all the components in the lowest level can be computed as shown in the example of the previous Section. These factors are then available for subsequent usage.

The next step is to compute, for all sparable parts, the change in readiness index due to an addition of one spare,

$$\Delta R_{lm}(n+1) \quad , \quad n=0,1,\cdots,n_{lm} \quad ,$$

which is necessary to establish the predetermined readiness for each sparable part. This provides a means of stopping the computations for ΔR_{lm}.

Once ΔR_{lm} is determined, the Essentiality,

$$E_{lm}(n+1) \quad , \; n=0,1,\cdots,n_{lm} \quad ,$$

can be computed. The Essentiality values are then ordered into descending order and, with a given constraint on total cost available

to establish an allowance list, spares are added in order of decreasing essentiality until the accumulated sum of costs is equal to, or is as close as possible to the constraint (if the constraint is active) or until the desired degree of Ships readiness is attained. It is assumed that the cost constraint will usually be active since, if a choice is available on maximum Readiness Index for the overall ship, it would be 1.0 .

In the above sparing scheme, it is understood that when a point is first reached where the addition of a spare violates the constraint, the next lower item is tested against the constraint. This procedure is followed until the constraint is met or the list is exhausted. Items for which there is no sparing in this scheme then contribute their installed readiness indices.

It must be pointed out that by using this procedure, a spare part which is added by virtue of high essentiality in a particular application is likely to be used in some other application of that spare for which no spare was provided. This situation could lead to serious consequences. However, no other particular allowance technique has adequately solved this problem. The old spare parts boxes used in the past would allow designation of particular applications but this is considered to be a step backward in view of the concept of central storage.

One possible method of accounting for this phenomenon would be to compute an "average" readiness R_s as follows:

$\Delta R_{1m}(n+1)$ is a constant for any of the m applications

of the lm^{th} sparable part due to the assumed identical and independent Poisson distribution. Each of the m applications will have a Transition Factor relating that application to ΔR_s. Therefore summing over the m applications

$$\sum_{i=1}^{m} (\Delta R_s)_i = \sum_{i=1}^{m} T_{N,i} f_{N,i} \Delta R_{lm}(n+1)$$

$$\sum_{i=1}^{m} (\Delta R_s)_i = \Delta R_{lm}(n+1) \left[\sum_{i=1}^{m} T_{N,i} f_{N,i} \right] ,$$

then dividing by m yields an "average" Transition function (not any longer a Transition Factor) which will yield a corresponding average ΔR_s.

This procedure ignores high essentiality applications and would probably yield a much lower Readiness Index.

Barlow and Proschan [1] have derived a procedure for determining the number of standby (spare) items necessary to optimize system reliability subject to a budgetary constraint. This procedure is applicable to series type models (METRI Series model is applicable), but there is no further development undertaken for other models. This concept could be further pursued.

It is to this point that the METRI Project had advanced. Further developments have not been considered.

8. Conclusions.

This thesis has attempted to take the METRI Interim Technical Report [2] and to clarify, simplify, and interpret some portions of that report which are thought to be vague, lack clarity, or a sound theoretical justification. It is believed that in trying to make the Series and Supplemental models extremely general and all-inclusive, the complicated notation used by the METRI authors made the understanding of these models overly difficult. More could have been gained by utilizing a simpler model.

The theoretical developments that were utilized in the METRI report were difficult to follow and in many cases important steps were omitted. Standard set theoretic notation is utilized in this thesis in order to facilitate the understanding of the reliability interpretations applied to the Alternate and Collateral Models.

It is here proposed that the clarification of the Basic Structure Models utilizing reliability theory has provided a sound mathematical basis for these models. The derivation of the Transition Factors is clearly stated and a step by step development is provided. In addition, a simple example is illustrated utilizing a complete structuring chart with a basic structure breakdown and the derivation of the Transition Factor.

There are many areas not covered in this thesis and one of the most important is the degrading factor, K_i, which is used in the Supplemental, Alternative, and Collateral models. In each case the K_i tend to have different meanings which certainly does not add

to clarity. In the Supplemental model it is meant to be only a weighting factor for different components of the structure. In the Collateral model it is a "nice to have" factor for nonessential elements which cause a decrease in reliability if not available but cannot cause failure. In the Alternative model it is a degrading factor for a piece of equipment not operating in its originally designed function, but which can operate as a substitute for some essential but nonoperative equipment.

A study [17] on derating factors has recently been made and perhaps an endeavor similar in nature could be investigated, in terms of the present problem.

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APPENDIX A

DERIVATION OF RELIABILITY EQUATIONS

Failures in complex systems are due to many and varied processes. The Wiebull distribution has been often used and displays three distinct areas of interest as follows: 1) the initial portion of the distribution demonstrates a decreasing failure rate indicative of a "wear-in or break-in" period, 2) the constant slope or failure rate portion of the distribution which covers a relatively large time period, and 3) the final portion of the distribution which is characterized by an increasing failure rate indicative of "wear-out or old age" failures.

Other sources of failures are the induced or secondary failures which occur due to some other part failing. The basic assumptions, as previously listed, exclude the secondary or induced failures. Further, the "burn-in, burn-out" failures are not considered directly. Reference [2] indicates that, since there are parts both installed and spared which are in various stages of their life cycle, we would expect all failure processes to be in evidence and we would therefore be concerned with predicting the average number of failures which will occur for a given type or configuration of parts. Clearly, this leads directly to considering an exponential failure distribution whose constant failure rate represents the average failure of the above distribution. This eliminates a considerably more complicated problem from an already sufficiently complicated situation.

The Poisson distribution follows directly when considering the

number of occurrences of exponentially distributed failures as can be readily verified in any basic probability text. Using a failure rate λ the parameter of the Poisson distribution becomes λt (the failure rate times a time period t). Using the Poisson distribution for a time period of T and a failure rate λ, it is known that the probability of exactly j failures occuring, where the failures are independent random events, is given by

$$P(j) = \frac{(\lambda T)^j e^{-\lambda T}}{j!} \quad , \quad j= 0,1,2,\cdots$$

Let there be m independent subsystems or applications which use the same component part, then, what is the probability that k failures occur, if K is the sum of N_i independent events, i.e.,

$$K= \sum_{i=1}^{m} N_i$$

where N_i is a counting function of the number of failures occuring in the i^{th} subsystem. The failures in each system are assumed to be Poisson (λT).

It is further known that the sum of independent Poisson distributed random variables is Poisson with the parameter equal to the sum of the individual parameters [18]. Since each N_i is independent and Poisson distributed, K is Poisson with parameter $m\lambda T$. Hence the probability of k occurrences in m systems is

$$P\left[K=k\right] = \frac{(m\lambda T)^k e^{-m\lambda T}}{k!} \quad , \quad k= 0,1,2,\cdots$$

Now consider m independent subsystems utilizing a particular

part. Each part has an exponential failure distribution with parameter λ. Let there be n identical spare parts in stock to be used to replace any of the m possible failures which might occur during the mission period T. Assuming that the time to install an available spare is negligible, it is desired to derive an expression for the probability, P_j, that the j^{th} $(j=1,2,\cdots,m)$ subsystem part will fail with no available spares during the mission T. Further consider the sequential and continuous time intervals as shown in Figure 7, such that

1) A is the event that n-1 failures occur during the interval (0,t)

2) B is the event that one failure occurs during the interval (t, t+dt)

3) C is the event that at least one failure occurs during the interval (T-t)

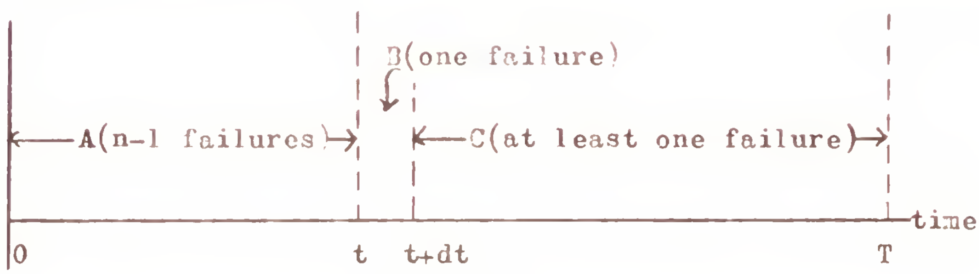

Figure 7.

the events A,B,C of Figure 7 are assumed to be independent, and hence

a) $$P\left[A\right] = \frac{(m\lambda t)^{n-1}\ e^{-m\lambda t}}{(n-1)!}$$

b) $$P[B] = m\lambda dt + o(dt)$$

where $$\frac{o(dt)}{dt} \to 0 \quad \text{as} \quad dt \to 0$$

c) $$P[C] = 1 - e^{-\lambda(T-t)}$$

Therefore, the probability (P_j) of the $(n+1)^{st}$ failure occuring in part j during the interval $(0,T)$ will be the product of events A, B, C, integrated over the interval $(0,T)$, i.e.,

$$P_j = \int_0^T \frac{(m\lambda t)^{n-1} e^{-m\lambda t} (1-e^{-\lambda(T-t)}) m\lambda dt}{(n-1)!}$$

$$= \int_0^T \frac{(m\lambda t)^{n-1} e^{-m\lambda t} m\lambda dt}{(n-1)!} - \int_0^T \frac{(m\lambda t)^{n-1} e^{-m\lambda t} e^{-\lambda(T-t)} m\lambda dt}{(n-1)!}$$

$$= \int_0^T \frac{m\lambda e^{-m\lambda t} (m\lambda t)^{n-1} dt}{(n-1)!} - e^{-\lambda T}\int_0^T \frac{(m\lambda t)^{n-1} e^{-\lambda(m-1)t} m\lambda dt}{(n-1)!}$$

multiplying the second term by $\left(\frac{m-1}{m-1}\right)^{n-1}$

$$= \int_0^T \frac{m\lambda e^{-m\lambda t} (m\lambda t)^{n-1} dt}{(n-1)!} - e^{-\lambda T}\left(\frac{m-1}{m-1}\right)^{n-1} m^n \int_0^T \frac{\lambda(\lambda t)^{n-1} e^{-\lambda(m-1)t} dt}{(n-1)!}$$

$$= \int_0^T \frac{m\lambda e^{-m\lambda t}(m\lambda t)^{n-1} dt}{(n-1)!} - e^{-\lambda T}\left(\frac{m}{m-1}\right)^n \int_0^T \frac{(m-1)\lambda e^{-\lambda(m-1)t} [(m-1)\lambda t]^{n-1} dt}{(n-1)!}$$

It can be shown [19] that each of the terms of the above expression is an Incomplete Gamma function and can be rewritten:

$$P_j = \sum_{k=n}^{\infty} \frac{e^{-m\lambda T}(m\lambda T)^k}{k!} - e^{-\lambda T}\left(\frac{m}{m-1}\right)^n \sum_{j=n}^{\infty} \frac{e^{-(m-1)\lambda T}\left[(m-1)\lambda T\right]^j}{j!}$$

$$= \sum_{k=n}^{\infty} \frac{e^{-m\lambda T}(m\lambda T)^k}{k!} - \left(\frac{m}{m-1}\right)^n \sum_{j=n}^{\infty} \left(\frac{m-1}{m}\right)^j \frac{(m\lambda T)^j e^{-m\lambda T}}{j!}$$

$$= \sum_{k=n}^{\infty} \frac{e^{-m\lambda T}(m\lambda T)^k}{k!} - \sum_{j=n}^{\infty} \left(\frac{m-1}{m}\right)^{j-n} \frac{e^{-m\lambda T}(m\lambda T)^j}{j!}$$

Therefore the reliability (readiness index of P_j), $R_j(n)$, (the readiness index of the j^{th} part due to n spares available) becomes

$R_j(n) = 1 - P_j = 1 - Pr\left[j^{th} \text{ subsystem part fails with no spares available} \right]$

$= 1 - Pr\left[n+1^{st} \text{ failure occurs} \right]$

$$= 1 - \left[\sum_{k=n}^{\infty} \frac{e^{-m\lambda T}(m\lambda T)^k}{k!} - \sum_{j=n}^{\infty} \left(\frac{m-1}{m}\right)^{j-n} \frac{e^{-m\lambda T}(m\lambda T)^j}{j!} \right]$$

$$= \sum_{k=0}^{n-1} \frac{e^{-m\lambda T}(m\lambda T)^k}{k!} + \sum_{j=n}^{\infty} \left(\frac{m-1}{m}\right)^{j-n} \frac{e^{-m\lambda T}(m\lambda T)^j}{j!}$$

Note that when j=n the first term of the second expression becomes

$$\frac{e^{-m\lambda T}(m\lambda T)^n}{n!}$$

which can be added to the first term as its n^{th} term so that the equation becomes

$$R_j = \sum_{k=0}^{n} e^{-m\lambda T} \frac{(m\lambda T)^k}{k!} + \sum_{j=n+1}^{\infty} \left(\frac{m-1}{m}\right)^{j-n} e^{-m\lambda T} \frac{(m\lambda T)^j}{j!}$$

Now to find ΔR_j when n+1 spares are available. As derived for the n^{th} spare, the R_j for the $n+1^{st}$ spare can be shown to be

$$R_j(n+1) = \sum_{k=0}^{n+1} e^{-m\lambda T} \frac{(m\lambda T)^k}{k!} + \sum_{j=n+2}^{\infty} \left(\frac{m-1}{m}\right)^{j-(n+1)} e^{-m\lambda T} \frac{(m\lambda T)^j}{j!}$$

therefore

$$\Delta R_j(n+1) = R_j(n+1) - R_j(n)$$

$$= \sum_{k=0}^{n+1} e^{-m\lambda T} \frac{(m\lambda T)^k}{k!} + \sum_{j=n+2}^{\infty} \left(\frac{m-1}{m}\right)^{j-(n+1)} \frac{(m\lambda T)^j e^{-m\lambda T}}{j!}$$

$$- \left[\sum_{k=0}^{n} e^{-m\lambda T} \frac{(m\lambda T)^k}{k!} + \sum_{j=n+1}^{\infty} \left(\frac{m-1}{m}\right)^{j-n} \frac{(m\lambda T)^j e^{-m\lambda T}}{j!} \right]$$

After simplification,

$$\Delta R_j(n+1) = e^{-m\lambda T} \frac{(m\lambda T)^{n+1}}{(n+1)!} - \left(\frac{m-1}{m}\right) e^{-m\lambda T} \frac{(m\lambda T)^{n+1}}{(n+1)!}$$

$$+ \sum_{j=n+2}^{\infty} \left(\frac{m-1}{m}\right)^{j-(n+1)} \frac{(m\lambda T)^j e^{-m\lambda T}}{j!} - \sum_{j=n+2}^{\infty} \left(\frac{m-1}{m}\right)^{j-n} \frac{(m\lambda T)^j e^{-m\lambda T}}{j!}$$

$$= \left[\frac{m}{m} e^{-m\lambda T} \frac{(m\lambda T)^{n+1}}{(n+1)!} - \left(\frac{m-1}{m}\right) e^{-m\lambda T} \frac{(m\lambda T)^{n+1}}{(n+1)!} \right]$$

$$+ \left[\sum_{j=n+2}^{\infty} \left(\frac{m-1}{m}\right)^{j-n} \left(\frac{m-1}{m}\right)^{-1} \frac{(m\lambda T)^j e^{-m\lambda T}}{j!} - \sum_{j=n+2}^{\infty} \left(\frac{m-1}{m}\right)^{j-n} \frac{(m\lambda T)^j e^{-m\lambda T}}{j!} \right]$$

$$= \frac{1}{m}\, \frac{e^{-m\lambda T}(m\lambda T)^{n+1}}{(n+1)!} + \sum_{j=n+2}^{\infty} \left(\frac{m-1}{m}\right)^{j-n} \frac{(m\lambda T)^{j}\, e^{-m\lambda T}}{j!} \left[\left(\frac{m-1}{m}\right)^{-1} - 1\right]$$

$$= \frac{1}{m}\, \frac{e^{-m\lambda T}(m\lambda T)^{n+1}}{(n+1)!} + \sum_{j=n+2}^{\infty} \left(\frac{m-1}{m}\right)^{j-n} \frac{(m\lambda T)^{j}\, e^{-m\lambda T}}{j!} \left[\frac{m}{m-1} - \left(\frac{m-1}{m-1}\right)\right]$$

$$= \frac{1}{m}\, \frac{e^{-m\lambda T}(m\lambda T)^{n+1}}{(n+1)!} + \sum_{j=n+2}^{\infty} \left(\frac{m-1}{m}\right)^{j-n} \frac{(m\lambda T)^{j}\, e^{-m\lambda T}}{j!} \left(\frac{1}{m-1}\right)\frac{m}{m}$$

whence

$$\Delta R_{pj}(n+1) = \frac{1}{m}\, \frac{e^{-m\lambda T}(m\lambda T)^{n+1}}{(n+1)!} + \frac{1}{m} \sum_{j=n+2}^{\infty} \left(\frac{m-1}{m}\right)^{j-n-1} \frac{(m\lambda T)^{j} e^{-m\lambda T}}{j!}$$

APPENDIX B

TOTAL DERIVATIVE EQUATIONS FOR METRI BASIC STRUCTURES

1. Series Model

$$R_u = \prod_{i=1}^{n} R_i$$

$$\frac{\partial R_u}{\partial R_j} = \prod_{i=1}^{n} \frac{R_i}{R_j}$$

$$dR_u = \sum_{j=1}^{n} \left(\prod_{i=1}^{n} \frac{R_i}{R_j} \right) dR_j$$

$$\therefore dR_u = \prod_{i=1}^{n} R_i \left(\sum_{j=1}^{n} \frac{dR_j}{R_j} \right)$$

2. Supplemental Model

$$R_u = \sum_{i=1}^{n} K_i R_i$$

$$\frac{\partial R_u}{\partial R_j} = K_j$$

$$\therefore dR_u = \sum_{j=1}^{n} K_j dR_j$$

3. Alternate Model

$$R_u = R_1 + K_2 R_2 \left[1 - R_1 \right]$$

$$\frac{\partial R_u}{\partial R_1} = 1 - K_2 R_2$$

$$\frac{\partial R_u}{\partial R_2} = K_2 - K_2 R_1$$

$$\therefore dR_u = (1 - K_2R_2)\ dR_1 + (K_2 - K_2R_1)\ dR_2$$

4. Collateral Model

$$R_u = R_1^{*}\left[K_2 + (1-K_2)\ R_2\right]$$

$$\frac{\partial R_u}{\partial R_1^{*}} = K_2 + (1-K_2)\ R_2$$

$$\frac{\partial R_u}{\partial R_2} = R_1^{*}\ (1-K_2)$$

$$\therefore dR_u = \left[K_2 + (1-K_2)\ R_2\right] dR_1^{*} + \left[R_1^{*}(1-K_2)\right] dR_2$$

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The "Military Essentiality Through Readiness Indices (METRI)" technique as presented in the Interim Technical Report to BuSandA, February 1965, was intended to describe the basic METRI model, the functional relationship between transition factors and the ship Readiness Index, as well as develop a technique for allowance list determination. However, many of the technical aspects of the report lacked clarity, some of the definitions were vague and doubly defined, and a sound theoretical justification was not adequately provided. The purpose of this thesis is to clarify and simplify certain technical aspects of METRI, provide theoretically sound models utilizing principles of reliability, and to derive the transition factors in a consistent and mathematically sound manner.

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14. KEY WORDS	LINK A		LINK B		LINK C	
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